Solution

We are tasked with evaluating:

$$\epsilon_{ijk}\{v_i, v_j\}_{P.B.}$$

for a non-relativistic particle of mass m and charge q moving in a magnetic field $B(r)$, where v denotes its velocity and $\{\cdot,\cdot\}_{P.B.}$ denotes the Poisson bracket.

Step 1: Expression for Velocity

The velocity v of a charged particle in a magnetic field is given by:

$$v=\frac{1}{m}(p-qA(r))$$

where:

- p is the canonical momentum,
- q is the charge of the particle,
- $A(r)$ is the vector potential related to the magnetic field by $B=\nabla\times A$.

Step 2: Poisson Bracket Calculation

The Poisson bracket between the components of velocity v_i and v_j is:

$$\{v_i, v_j\}=\left\{\frac{1}{m}(p_i-qA_i), \frac{1}{m}(p_j-qA_j)\right\}$$

Using the properties of Poisson brackets and the fact that $\{p_i, p_j\}=0$, $\{p_i, A_j\}=\partial_i A_j$, and $\{A_i, A_j\}=0$, we compute:

$$\{v_i, v_j\}=\frac{q}{m^2}(\partial_i A_j-\partial_j A_i)$$

This expression corresponds to the components of the magnetic field:

$$\{v_i, v_j\}=\frac{q}{m^2}B_k\epsilon_{ijk}$$

Step 3: Final Expression

Multiplying by ϵ_{ijk}, we obtain:

$$\epsilon_{ijk}\{v_i, v_j\} = \frac{q}{m^2} B_k$$

Conclusion

The correct answer is:

$$\frac{q}{m^2} B_k$$